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Measuring the Technical Efficiency of Hockey Legends: Who is the Best NHL Player of All Time?

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ABSTRACT

The aim of the research is to determine the technical efficiency of legendary hockey players in the National Hockey League (NHL), to create a ranking of these players and to reveal the best NHL players of all time. The research uses statistical data on 379 players from the 1944/45 season to the 2023/24 season. The methodology is based on multi-criteria analysis, specifically the concept of data envelopment analysis (DEA). The DEA provides an objective measure of the overall playing profile of hockey players and can help supplement the information in rankings provided by sports journalists. Andersen and Petersen's model is used to evaluate the data collected, providing super efficiency scores by aggregating NHL statistics related to various aspects of the game to produce a final ranking of hockey legends. The concept of data envelopment analysis works with multiple variables and allows for greater objectivity to be incorporated into the rankings. The number of games played is chosen as one of the model's input variables. The output variables include: number of goals scored, number of assists, plus/minus, inverse of penalty minutes, points per game, number of shots, number of individual awards, and number of Stanley Cups won. The research named Wayne Gretzky, Butch Goring and Serge Savard as the best players in the NHL historically in terms of technical efficiency. Among other things, the ranking of legendary hockey players revealed that players with a high number of games played or points scored are not necessarily technically efficient.

Keywords: sport; ice hockey; technical efficiency; National Hockey League; DEA

INTRODUCTION

The question of which players are historically the best in various sporting competitions is much more than a philosophical consideration. Fans, media and the general public often wonder who is the best athlete in a given year or throughout history in a given sport. But the best athletes can also be evaluated from a national or club level perspective. Every year the media and sports journalists compile various rankings of the best athletes. The main problem with these rankings can be that they are too subjective. Rarely is one final indicator created that sums up all aspects and contributions of a given athlete (DeOliveira and Callum, 2004). Each country may also take into account different factors, have different voting rules, and some polls involve fans. The focus of sports journalists who vote on athletes also plays a big role. The popularity of the sport in question is also a major factor. Last but not least, the media image that athletes create about themselves also plays a role. The media image of athletes is influenced by a number of factors. These factors include qualitative factors in the form of personal charisma, and a life attitude that sets an example not only to teammates but also to people in their environment (Agrawal and Kamakura, 1995; Pelloneová, 2023b). In addition to these factors, sports journalists may also consider individual player statistics in the form of number of goals scored, number of assists, number of shots or number of games played when compiling rankings (Weeks, 2021).

Traditionally, hockey players are evaluated based on statistical indicators, including goals, assists, points (sum of goals and assists) and plus/minus statistics (difference between goals scored and goals allowed on the ice). At the same time, other more sophisticated data can be used that goes into more detail about how players are doing in terms of puck possession. This paper proposes a methodology using data envelopment analysis (DEA) that takes into account heterogeneous inputs and outputs to calculate which players are technically efficient. Measuring technical efficiency is a frequently used method in sports. In particular, technical efficiency has been measured when evaluating a sports team as a whole (Guzmán and Morrow, 2007; Djordjević et al., 2015; Ribeiro and Lima, 2012; Haas, 2003). Only a limited number of researchers have investigated the technical efficiency of individual athletes and used it to subsequently create their rankings. For example, basketball players are ranked by Chen et al. (2016). Rankings of the best golfers were produced by Fried et al. (2004) and Alcaraz et al. (2019). American baseball players are covered by Chiu et al. (2015) and Mazur (1994). Rankings of the world's best tennis players were created by Ruiz et al. (2011), Ramón et al. (2012), and Halkos and Tzeremes (2012). Most authors have focused on football players (Kapera, 1996; Alp, 2006; ul Haq Bhat et al., 2021; Tiedemann et al., 2011; Hirotsu et al., 2018). For example, Santín (2014) uses a super efficiency model in order to identify the historically best players of Real Madrid club. Pelloneová and Tomíček (2022a) use a DEA model to rank the best players in the Czech and Danish football leagues. Further research by Pelloneová and Tomíček (2022b) evaluates goalkeepers of the first and second Slovak football leagues using an input-oriented DEA model and Andersen-Petersen super efficiency model. The field of hockey is addressed in the research of Pelloneová (2023a), where the technical efficiency of hockey players playing in the Czech top hockey competition is evaluated. Pelloneová (2023b) research is devoted to NHL players, ranking the best Czech forwards, defenders and goalkeepers. Next, Vavrek (2021) then uses multi-criteria decision-making to rank the hockey players for the 2018/19 season.

Based on the above literature search, it is clear that the concept based on operational analysis has been used in many sports. Research in the field of NHL is not widespread in the foreign literature. The popularity of the NHL in Europe is at a fairly high level (Langr, 2019). On the other hand, in the US, the NHL has the lowest viewership of the four major sports leagues, with 17% of American sports fans following the NHL, according to a survey by Ampere Sports Consumer (2022). Despite this, it is still the best hockey league in the world according to Helfrick (2024). Sportswriters very often rank the NHL's all-time best hockey players. There are several rankings of the most successful hockey players available. For example, according to Bharnuke (2023), the best hockey player is Wayne Gretzky followed by Mario Lemieux and Bobby Orr. According to Kenyon (2018), Gordie Howe is at the top, followed by Wayne Gretzky and Bobby Orr. According to Duffett (2023), the best players in NHL history are Wayne Gretzky, Bobby Orr and Mario Lemieux. In addition to the aforementioned players, Mark Messier, Ron Francis, Ray Bourque, Joe Thornton, Marcel Dionne, etc. also appear in the top ten. However, these rankings take into account mainly the number of points scored for goals and assists. Ranking the NHL's best players historically is not an easy task and it is likely that bias will enter into the rankings. Currently, there is a lack of a comprehensive methodology that incorporates multiple factors into the selection of the best players at the same time and is independent of subjective evaluation. The proposed approach for measuring the technical efficiency of hockey legends is based on the use of multi-criteria analysis, specifically the concept of data envelopment analysis. The use of data envelopment analysis in the evaluation of technical efficiency allows for the examination of several variables affecting the resulting player efficiency and brings greater objectivity to the evaluation. A basic input-oriented model with constant returns to scale and the Andersen and Petersen model were used to analyse efficiency in order to rank NHL hockey legends and identify the best players historically.

METHODS AND DATA

Data

The subject of the present research was a dataset containing a list of hockey legends playing in the NHL, the world's most popular hockey competition. The research focused only on hockey players who played more than 1,000 regular season games in the NHL. Playoff games were excluded from the research. These were hockey players who had played a large enough number of games to be considered legends. At the same time, due to the large number of games played, it is possible to reliably observe their sports performance. The created file contains a total of 379 field players (skaters). The research did not include goalkeepers, as only four players (Martin Brodeur, Patrick Roy, Roberto Luongo and Marc-Andre Fleury) exceeded 1,000 games played and goalkeepers require different metrics to be tracked. Among the 379 hockey players evaluated, there were a total of 124 defenders (D), 110 forwards playing the center position (C), 65 forwards playing the left wing position (L) and 80 forwards playing the right wing position (R).

These are hockey players who played in the NHL from the 1944/45 season to the 2023/24 season. The database includes notable players such as Claude Provost, Jaromir Jagr, Gordie Howe and Wayne Gretzky. The database also includes hockey players who are active in the NHL at the time of the research. For example, in the 2023/24 season, these are: Alex Ovechkin, Sidney Crosby, Evgeni Malkin, Anze Kopitar, Nicklas Backstrom and others. The majority of the players were Canadian (69.7%) followed by Americans (15.6%), Swedes (4.2%), Czechs (3.4%), Finns (2.1%), Russians (2.1%) and Slovaks (1.8%). Austria, Lithuania, Serbia and Slovenia were represented by one player each.

Data on individual players was obtained from publicly available data in the form of official statistics on the website of the American ice hockey league NHL (2024). Both performance data in the form of individual game statistics and individual and team awards were selected for the research. From the category of individual game statistics, the research included the number of games played, number of goals scored, number of assists, plus/minus, penalty minutes, points per game and shots. Other interesting individual statistics such as the number of hits, the number of shots blocked by a player, takeaways or faceoff percentage could not be included in the research. The statistics on face-off percentage were only tracked from the 1997/98 season onwards. The statistics of hits, shots blocked by a player and takeaways were not tracked in the NHL until the beginning of the 2005/06 season. Thus, this data would not have been available for approximately 30% of the players and therefore was not included in the research. From the individual and team awards category, the number of individual trophies and the number of Stanley Cups were selected. The individual awards category included a number of trophies e.g. Bill Masterton Memorial Trophy, King Clancy Memorial Trophy, NHL Foundation Player Award, Conn Smythe Trophy, James Norris Memorial Trophy, Lady Byng Memorial Trophy, Lester Patrick Trophy, Art Ross Trophy, Calder Memorial Trophy, Hart Memorial Trophy, Maurice “Rocket” Richard Trophy, Ted Lindsay Award. These trophies reflect not only the different performance skills of the players but also their character traits. Included here are awards such as the NHL most valuable player, the award for the most points in the regular season, the defenseman award, the award for outstanding sportsmanship and gentlemanly conduct combined with playing ability, and the award for the player who has brought the core values of hockey into the lives of others. The most famous team award is the Stanley Cup. The first time the trophy was awarded was in 1893, with the NHL winner receiving the trophy.

Data Envelopment Analysis

The present paper uses one of the methods of quantitative analysis to evaluate technical efficiency. The technical efficiency of NHL hockey legends was evaluated by data envelopment analysis. Data envelopment analysis uses mathematical programming techniques, is used to evaluate the efficiency of homogeneous decision making units (further DMUs), and determines the relative degree of technical efficiency. Data envelopment analysis is classified as a multi-criteria decision-making model because it identifies efficient and inefficient DMUs by evaluating a number of different types of inputs and outputs. The efficiency of units is determined by optimizing the weights of inputs and outputs in the technical efficiency coefficient. The calculated technical efficiency is then dependent on the number of production units evaluated and the number and structure of inputs and outputs (Dlouhý et al., 2007; Banker et al., 1984). A DMU can be considered efficient if it consumes a small amount of inputs in relation to the production of a large amount of outputs. A unit is efficient if the technical efficiency coefficient is equal to one.

Many methods have been developed and modified to assess technical efficiency. Among the most important models are the radial models by Charnes, Cooper and Rhodes (CCR model) and Banker, Charnes and Cooper (BCC model). Both models include radial variables that specify the minimum required reduction rate of all inputs and the rate of increase of all outputs to reach the efficient frontier (Charnes et al., 1978; Banker et al., 1984). These models must distinguish between input-oriented versus output-oriented versus additive models. By using the above models, it is possible not only to obtain an estimate of the technical efficiency rate of DMUs and to organise them on the basis of this rate, but also to provide information on how their behaviour should be improved to make these evaluated DMUs efficient. According to Ramanathan (2003) the choice of model is primarily at the discretion of the evaluator, although some authors provide some recommendations that should be followed in their choice. In this research, a radial input-oriented model considering constant returns to scale (CCR model) was chosen based on the recommendations of authors (Jablonský and Dlouhý, 2015; Cooper et al., 2007; Jablonský et al., 2018) and previous experience (Pelloneová, 2023b; Jablonský, 2021). The dual formulation of this model is captured by relation (1). Where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ is the vector of weights assigned to each DMUs, s_k^+, $k = 1, \ldots, r$, and s_j^-, $j = 1, \ldots, m$ are vectors of additive variables in the input-output constraints, θq is a scalar variable that measures the required reduction of inputs to reach the efficient frontier (a measure of the technical efficiency of q-th DMU) and ε is an infinitesimal constant (Dlouhý et al., 2007; Jablonský et al., 2018). The result is the computed overall technical efficiency score (further OTE score).

$$\begin{aligned}
&\min \theta_q - \varepsilon\left(\sum_{j=1}^{m} s_j^- + \sum_{k=1}^{r} s_k^+\right),\\
&\sum_{i=1}^{n} x_{ij}\lambda_i - s_j^- = \theta_q x_{qj}, j = 1, \ldots, m,\\
&\sum_{i=1}^{n} y_{ik}\lambda_i - s_k^+ = y_{qk}, k = 1, \ldots, r,\\
&s_k^+ \geq 0, k = 1, \ldots, r,\\
&s_j^- \geq 0, j = 1, \ldots, m,\\
&\lambda_i \geq 0, i = 1, \ldots, n,\\
&\sum_{i=1}^{n} \lambda_i \in R
\end{aligned} \qquad (1)$$

Radial DEA models do not allow further classification of efficient units. Super-efficiency models eliminate this shortcoming. Depending on the type of model chosen, on the relationship between the number of DMUs and the number of inputs and outputs, there may be a larger number of efficient DMUs that need to be ordered. The most well-known super-efficiency models include the Andersen and Petersen model, slacks-based measure of super-efficiency model (S-SBM model) developed by Tone (2002) and super-efficiency goal programming model (SBMG model) developed by Jablonský (2012). The research used the most widely used super-efficiency model by Andersen and Petersen (1993) with an input orientation. This model differs from the radial CCR model only by the added condition $\lambda q = 0$. If the extracted unit Uq is efficient, the efficient frontier is modified. Subsequently, a new unit is sought U_q^* that is on the production frontier given the new set of DMUs. The efficiency measure is calculated as the distance between the units Uq and U_q^*. The super-efficiency measure θ_q^{AP} is ≥ 1 for efficient units and the same for inefficient units as in the CCR model. The higher the value of θ_q^{AP} the higher the position of the DMU in the overall ranking. Mathematically, the Andersen and Petersen input-oriented model assuming constant returns to scale can be expressed according to Andersen and Petersen (1993) using relation (2). The result is the calculated AP score. The MaxDEA Ultra software developed by Beijing Realworld Research and Consultation Company Ltd (2024) was used for all the calculations performed.

$$\begin{aligned}
&\min \theta_q^{AP},\\
&\sum_{i=1}^{n} x_{ij}\lambda_i + s_j^- = \theta_q^{AP} x_{qj}, j = 1, \dots, m,\\
&\sum_{i=1}^{n} y_{ik}\lambda_i - s_k^+ = y_{qk}, k = 1, \dots, r,\\
&s_k^+ \geq 0, k = 1, \dots, r,\\
&s_j^- \geq 0, j = 1, \dots, m,\\
&\lambda_i \geq 0, i = 1, \dots, n, i \neq q,\\
&\lambda_q = 0,\\
&\sum_{i=1}^{n} \lambda_i \in R
\end{aligned} \tag{2}$$

Using an input-oriented DEA model based on constant returns to scale, efficient and inefficient players were identified based on OTE scores. Then, using the Andersen and Petersen model, AP super efficiency scores were calculated and a final ranking of efficient hockey legends was generated. Both DEA models worked with one input variable, which was the number of NHL games played (GP). On the other hand, the research included eight output variables. The following output variables were included in the research: the number of goals scored (G), the number of assists (A), the plus/minus (+/-), the inverse of the number of penalty minutes (P), the number of points per game (P/GP), the number of shots (S), the number of individual awards (T), and the number of Stanley Cups (Sc).

The above variables were selected based on three conditions. The first condition was the availability of all selected statistics for all evaluated players. The second condition took into account the Pearson correlation coefficient (*r*) of the selected outcome variables. With a very strong correlation ($r > 0.8$), the resulting effect of the included outcome factor could be duplicated. Therefore, it would be inappropriate to include an output variable in DEA models that is very strongly correlated with all other output variables. Table 1 shows the correlations of the examined output variables. The strongest correlation was between the number of goals scored and the number of points per game ($r = 0.89$), followed by the number of assists and the number of points per game ($r = 0.87$), and then the number of goals scored and the number of shots ($r = 0.83$). Due to the fact that a very strong correlation was found only with a maximum of two variables and not with all of them, these variables were retained in the research. A strong correlation was found between the three variables ($0.60 \leq r \leq 0.79$). A relatively weak correlation was found especially between the number of team trophies and individual game statistics.

Table 1. Correlation matrix of variables (Pearson correlation coefficients)

	Stanley Cups	Trophies	Goals	Assists	Plus-Minus	Penalty minutes	Points per game played	Shots
Stanley Cups	–							
Trophies	0.27	–						
Goals	0.12	0.50	–					
Assists	0.12	0.46	0.70	–				
Plus-Minus	0.42	0.42	0.22	0.35	–			
Penalty minutes	0.05	0.04	0.08	0.06	-0.01	–		
Points per game played	0.13	0.53	0.89	0.87	0.32	0.13	–	
Shots	0.04	0.47	0.83	0.72	0.25	0.00	0.77	–

The third condition was to determine the optimal number of input and output variables given the number of hockey players being evaluated. When it comes to the number of DMUs (i.e., the size of the sample under study), it is advantageous to have a larger data set because, for a given number of DMUs, the efficiency outcome of each DMU may depend on the number of variables (Cinca and Molinero, 2004). As stated by Jenkins and Anderson (2003) the larger the number of variables, the less informative the DEA analysis. The literature presents some empirical rules regarding the number of DMUs compared to the number of inputs and outputs. The number of DMUs evaluated should be at least twice the number of inputs and outputs (Golany and Roll, 1989; Homburg, 2001). On the other hand, a number of other authors (e.g., Nunamaker, 1985; Banker et al., 1989; Friedman and Sinuany-Stern, 1998; Raab and Lichty, 2002) recommend even three times the number of variables relative to the number of DMUs. Another empirical rule is relation 3 of Cooper et al. (2007), where n is the number of DMUs, m is the number of input variables and s is the number of output variables.

$$n \geq \max\{m \times s, 3 \times (m + s)\} \quad (3)$$

Given the 379 DMUs evaluated and the nine variables included in the DEA models, all the rules described above are satisfied.

RESULTS

This part of the article is devoted to the results of empirical research. An input-oriented CCR model was applied to a set of 379 field players (skaters) and their selected variables to calculate an overall technical efficiency score (OTE score). The average OTE score reached relatively high values around 0.601. The calculated median OTE score is 0.602, based on this obtained OTE score value it can be stated that half of the evaluated hockey legends achieved worse OTE score values and half achieved better values. Out of 379 hockey legends evaluated, 16 players were found to be efficient in terms of the calculated technical efficiency score. An input-oriented Andersen and Petersen model was used to calculate the super-efficiency scores to determine the ranking of the efficient hockey legends. Table 2 shows the efficient hockey legends ranked by AP score. From Table 2, it can be seen that the most important NHL hockey legend is Wayne Gretzky. Wayne Gretzky is not the record holder for most games played. Patrick Marleau played the most games in the NHL, ranking 214th with an OTE score of 0.573. However, Wayne Gretzky is the player who has scored the most goals and had the most assists in NHL history. Wayne Gretzky is also among the eight highest scoring players in the NHL. He also earned the most points per game (1.92) of all 379 players rated. He ranked third in individual plus/minus statistics. In terms of penalty minutes, he was ranked 276th with 577 penalty minutes, making him one of the less penalized player. He won the Stanley Cup four times during his NHL career. Those seasons were 1983/84, 1984/85, 1986/87 and 1987/88 with the Edmonton Oilers. He also won the most individual trophies and awards of any player evaluated. Butch Goring was the second highest rated. Butch Goring played in 1,107 games, scoring 888 points on goals and assists. He won the Stanley Cup with the Los Angeles Kings and New York Islanders from 1979/80 to 1982/83. Third overall was defender Serge Savard, who earned 439 points for goals and assists in 17 seasons in the NHL. As a defender, he scored fewer goals than other efficient players. On the other hand, he had a pretty favorable plus/minus statistic. He won a total of eight Stanley Cups with the Montréal Canadiens. The overall ranking of other efficient hockey legends can be seen in Table 2.

The ranking of 16 technically efficient hockey legends also shows that almost all players are Canadian. The only exception is the Russian player Alex Ovechkin, who is the only technically efficient player still active in hockey. The oldest player is Jean Beliveau, who was born in 1931. On the other hand, the youngest hockey legend is the aforementioned Alex Ovechkin. Table 2 shows that among the technically efficient hockey legends all player positions are included. Hockey legends include mainly forwards (12 technically efficient players). The most technically efficient players were at the center position (6 technically efficient players in total). On the left and right wings there were three players marked as efficient. Table 2 shows that all technically efficient players have won individual trophies during their hockey careers. Almost all of the technically efficient players also won the Stanley Cup group trophy. The only exceptions are forwards Rick Middleton and Craig Ramsay. The final standings of the other 363 players are not included in this article due to space limitations.

Table 2. Ranking of efficient hockey legends and values of variables

R	Player	Pos	AP score	GP	G	A	+/-	P	P/GP	S	Sc	T
1.	W. Gretzky	C	1.983	1,487	894	1,963	520	0.0017	1.92	5,088	4	32
2.	B. Goring	C	1.331	1,107	375	513	-26	0.0098	0.8	2,275	4	3
3.	S. Savard	D	1.243	1,040	106	333	462	0.0017	0.42	1,390	8	2
4.	J. Beliveau	C	1.234	1,125	507	712	117	0.0010	1.08	2,627	10	5
5.	L. Robinson	D	1.175	1,384	208	750	722	0.0013	0.69	2,332	6	3
6.	A. Ovechkin	L	1.173	1,347	822	663	67	0.0013	1.1	6,355	1	18
7.	G. Lafleur	R	1.139	1,126	560	793	446	0.0025	1.2	3,516	5	9
8.	C. Provost	R	1.094	1,005	254	335	101	0.0021	0.59	2,175	9	1
9.	R. Middleton	R	1.090	1,005	448	540	175	0.0064	0.98	2,272	0	1
10.	B. Hull	L	1.082	1,063	610	560	249	0.0016	1.1	4,577	1	7
11.	H. Richard	C	1.078	1,258	358	688	243	0.0011	0.83	3,199	11	1
12.	D. Keon	C	1.068	1,296	396	590	61	0.0085	0.76	3,946	4	4
13.	R. Bourque	D	1.038	1,612	410	1,169	527	0.0009	0.98	6,209	1	8
14.	C. Ramsay	L	1.035	1,070	252	420	324	0.0050	0.63	1,617	0	1
15.	D. Potvin	D	1.020	1,060	310	742	456	0.0007	0.99	3,050	4	4
16.	B. Clarke	C	1.020	1,144	358	852	507	0.0007	1.06	2,582	2	7

R = ranking, Pos = game post, D = defender, C = center, L = left wing, R = right wing, AP = Andersen-Petersen, GP = games played, G = goals, A = assists, +/- = plus/minus, P = inverse of the number of penalty minutes, P/GP = points per game, S = shots, Sc = Stanley Cup, T = individual awards.

Among the players who have played more than 1,000 games in the NHL are 13 Czech players. However, none of them was named a technically efficient player. Of the Czech players, Jaromir Jagr ranked best in the overall Hockey Legends Ranking, coming in 55th according to OTE score (0.801). Gordie Howe achieved a rather surprising ranking. Gordie Howe appears very often in rankings of the NHL's top 10 players. He was ranked as high as 74th in technical efficiency with an OTE score of 0.763. Nicklas Lidstrom achieved a similar ranking, coming in 75th. Another famous and still active hockey player, Sidney Crosby, ranked 29th according to his OTE score (0.894).

In total, 363 players (95.78%) can be classified as inefficient units. Relatively good OTE scores (> 0.9) were achieved by players Brett Hull (0.999), Phil Esposito (0.970) and Marcel Dionne (0.963). These players ranked 17th to 19th in the rankings. Further calculations revealed that out of 363 inefficient players, 11 players had scale inefficiency. Thus, the main source of their inefficiency was an incorrectly chosen game strategy of the whole team. None of the players had pure technical inefficiency as the source of their inefficiency. For the remaining 352 players, the main source of inefficiency was overall technical inefficiency. Thus, these players had problems with both game performance and choice of game strategy. Figure 1 compares the sources of inefficiency for each type of playing position.

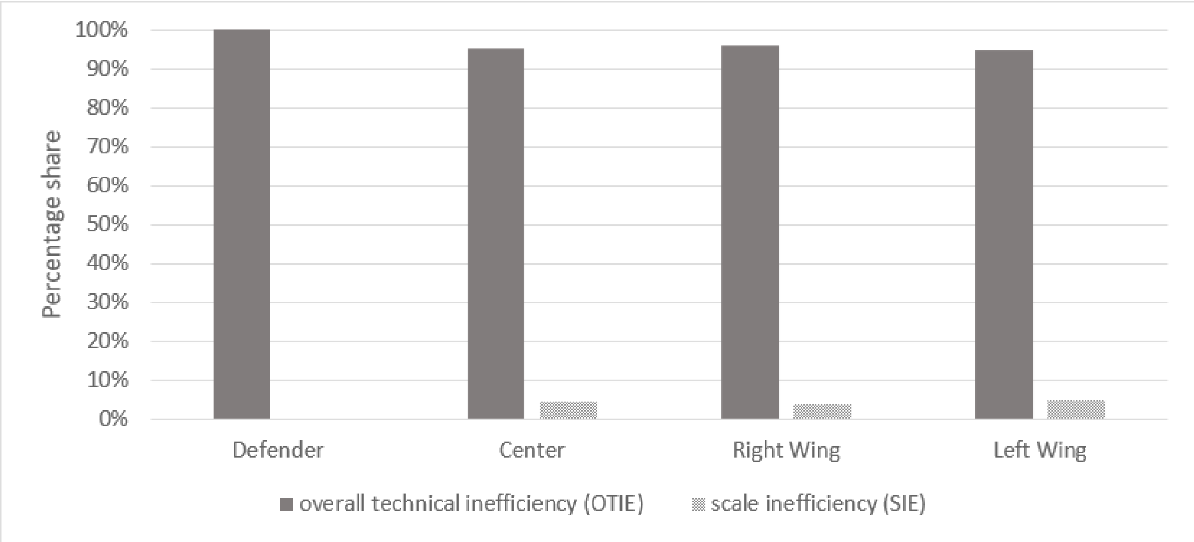

Figure 1. Comparison of sources of inefficiency within individual game positions

Figure 1 shows that scale inefficiency was not a common problem for hockey legends. It appeared only in offensive posts, but not to a greater extent than in 5% of cases. This source of inefficiency is not influenced by the playing skills of the player but rather the game strategy of the entire team. For all playing positions, overall technical inefficiency prevailed. Thus, for most players, inefficiency was found to be primarily related to individual playing skills.

DISCUSSION

Using methods from the field of operational analysis, it is possible to create a definitive ranking of hockey legends. The resulting ranking differs in many aspects from the ranking produced by sports journalists. The only hockey legend that most available rankings dedicated to the NHL (Duffett, 2023; Bharnuke, 2023; Kvasnička, 2022), including this research, agree on is hockey legend Wayne Gretzky. Wayne Gretzky is widely regarded as the greatest hockey player of all time, and this research confirms this by data envelopment analysis. His skills, records, numerous accolades and huge impact on the sport make him a legendary figure in hockey history. Wayne Gretzky holds 61 NHL records, among others. Wayne Gretzky popularized hockey and reached fans around the world. He became a cultural icon and brought a new level of excitement to the sport. Duffett (2023), providing a ranking of the NHL's top 25 hockey players, also named Jean Beliveau as a notable player and ranked him 7th overall. Alex Ovechkin, Guy Lafleur, Brett Hull and Ray Bourque were also included in this ranking. However, in the translated survey they all achieved a better position than in the ranking by Duffett (2023).

Research has also revealed that players with a high number of games played are not necessarily technically efficient. Such as legends Patrick Marleau, Gordie Howe, Mark Messier, Jaromir Jagr or Ron Francis, who have played more than 1,700 games in the NHL. Nor is the plus/minus statistic the deciding factor. Plus/minus is the official statistic of hockey competitions and is one of the most popular ones, especially in the Czech Republic. It has been proven more than once (Deshpande and

Jensen, 2016) that evaluating any player based on plus/minus often leads to wrong conclusions. The NHL still records the plus/minus statistic for players, but it is no longer very attractive among analysts and coaches. For example, the data envelopment analysis identified Butch Goring as a technically efficient player despite an unfavourable value of -26. The deciding factors for selecting the best players are not necessarily individual or team trophies. The only exception to the number of individual trophies is Wayne Gretzky, who has collected the most individual awards during his career. On the other hand, players like Gordie Howe, Sidney Crosby, Jaromir Jagr and Phil Esposito, who hold more than 10 individual trophies, have been labeled as technically inefficient units. In terms of team valuation, winning the Stanley Cup is the most important factor. Henri Richard holds the NHL record for most Stanley Cups. From 1956 to 1973 he won 11 Stanley Cups, all with the Montréal Canadiens hockey team. The DEA's method of ranking this player as technically efficient ranked him 11th overall. Jean Beliveau, who's won 10 Stanley Cups, finished 4th in the AP score rankings. Still, the number of team trophies may not be the only deciding factor. A combination of individual game stats and individual awards and club success should be the deciding factors in selecting the best players.

The research presented here is limited to players who have played more than 1,000 games in the NHL. Thus, hockey legends such as Mario Lemieux (played 955 games) or Bobby Orr (played only 657 games) were not included. Although this research would have liked to examine all types of players, goalkeepers were left out of the research because their metrics are different from those of field players and would have warranted a separate study. Since there are more skaters than goalkeepers, more data is available and it makes sense to start with them. The research presented here is also dependent on data availability. Some variables could not be included in the research because the NHL did not start tracking them until later seasons and they were not available for all players.

The results show that DEA models, and multi-criteria decision-making methods in general, provide a comprehensive multidimensional approach to assessing the technical efficiency of hockey players. The DEA methodology is quite different from the conventional methods used by sports journalists to rank players. . They most often compile rankings based on one variable in the form of player productivity (the number of so-called Canadian points for goals scored and assists). Multi-criteria decision-making methods allow a relatively wide range of variables to be included in the rankings. The DEA method also has the advantage of not requiring a priori weighting of the relevant input-output factors. The DEA method is flexible and allows a large number of input-output factors to be taken into account, which may be both quantitative and qualitative in nature. DEA has been used quite frequently in various sporting disciplines, but only a minimum of researchers have used it in hockey, which was the motivation for this study.

CONCLUSION

The paper focused on the evaluation of the technical efficiency of NHL hockey legends. The aim was to create a ranking of legendary NHL players and to identify technically efficient players who can be considered the best in terms of multi-criteria decision-making. A radial input oriented data envelopment analysis model operating under the assumption of constant returns to scale was used to determine the technical efficiency scores. The Andersen and Petersen super efficiency model was used to generate the final ranking of the best hockey players in NHL history. DEA analysis helped with the creation of the player rankings and identified 16 players as the best historically in terms of technical efficiency. Hockey legend Wayne Gretzky was ranked first, Butch Goring second and Serge Savard third. Future research could focus on comparing the results of different super efficiency methods (e.g. S-SBM). Also, other sets of input and output variables could be chosen for future analysis, such as number of wins in international tournaments (Olympics, World Cup of Hockey, etc.), NHL contract value, etc.

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