Measuring the Technical Efficiency of Hockey Players: Empirical Evidence from Czech Hockey Competition

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ABSTRACT

Ice hockey is a very popular sport in the Czech Republic. Nowadays, hockey player efficiency analysis is a useful tool that helps sports managers with player selection, team composition and team performance evaluation. The literature offers only a limited number of scientific studies that deal with the evaluation of the efficiency of hockey players or clubs. The aim of this research is to use data envelopment analysis to help Czech hockey clubs, managers and coaches to evaluate the efficiency of their players. This research evaluates the technical efficiency of Czech hockey players using three data envelopment analysis models, ranks the best players based on their super-efficiency scores, and then tries to uncover the main sources of player inefficiency. The models are empirically applied to players playing in the Tipsport extraliga in the 2021/22 season. The evaluation used in this paper attempts to incorporate greater objectivity into decision making and thus may be an important step in developing a systematic methodology for evaluating hockey players.

Keywords: sport; ice hockey; efficiency; inefficiency; data envelopment analysis

INTRODUCTION

Efficiency is a performance criterion for any organizational system based on the quantity of inputs and outputs (Kian, 2009). In this research, efficiency is defined as the ratio of outputs to inputs needed to produce those outputs. In the current economically challenging times, sports clubs need to use their resources judiciously and pay attention to measuring their efficiency. Due to the nature of sports clubs, it is often quite difficult to measure their efficiency and performance. Sports clubs are a special kind of business because, although they operate within the same legal framework as other companies, they are very strongly conditioned by sporting activities. Every sports organization in the current adverse times needs a comprehensive system to measure its efficiency to be aware of the desirable and undesirable elements in its operations. The sports industry has been undergoing a development in the last two years, especially economically. Sports competitions around the world have been negatively affected by the global Covid-19 pandemic in 2020 and 2021. Czech sports clubs are currently struggling with high energy prices and a post-viral decline in spectator attendance. Every sports organisation should therefore try to assess its strengths and weaknesses. Success in a professional sports league today goes hand in hand with a successful team, coach and ultimately team leadership. The focus should be on determining whether the sports club is using its human resources appropriately to achieve the best possible sporting results. It is conceivable that a club whose players are relatively cheap can get "interesting" sporting results. Evaluating the efficiency of individual players is currently a key issue in the sports industry. Because the success of the club and the success of the team is built primarily on individual players. This raises the question of how to measure the sporting efficiency of players.

Efficiency measurement in professional sport has an extensive list of empirical analyses in the sport economics literature. The following list contains only a few examples from different sports. These are mainly foreign researches focusing on football (Collier et al., 2011; Hadley et al., 2000; Haas, 2003; Haas et al., 2004; Guzmán and Morrow, 2007; Espitia-Escue and García-Cebrián, 2004; Palafox-Alcantar and Vargas-Hernández, 2015; Zambom-Ferraresi et al., 2019; Guzmán-Raja and Guzmán-Raja, 2021; Kirschstein and Liebscher, 2018), basketball (Zak et al., 1979; Fizel and D'Itri, 1996; Cooper et al., 2009; Cooper et al., 2011), baseball (Porter and Scully, 1982; Ruggiero et al., 1996; Andersen and Sharp, 1997; Mazur, 1994; Suk, 2014), golf (Fried et al., 2004) and tennis (Halkos and Tzeremes, 2012; Ramón et al., 2012). Most of the aforementioned efficiency studies evaluate the technical efficiency of sports clubs or teams as a whole. Despite numerous studies conducted in many different countries, few studies have attempted to include measures of the technical efficiency of individual players. The player is an important element behind the success of the team and the sports club. The literature review further revealed that there is almost no attention paid to ice hockey among researchers. Ice hockey is a very popular sport worldwide nowadays. The hockey industry is a major industry in many countries. Ice hockey has a long history and tradition in the Czech Republic and its popularity is at a high level. Therefore, efficiency analysis is as important in ice hockey as in other sports.

The literature review further revealed that operational research techniques are mainly used to measure efficiency in professional sport. One of the most commonly used techniques is data envelopment analysis (further DEA). The following section briefly summarizes the authors who apply data envelopment analysis or other operational research techniques in their research to evaluate the efficiency of hockey clubs or individual players. These are mainly authors who evaluate the efficiency of clubs from the National Hockey League (further NHL). Kuosmanen (1998) evaluates the efficiency of NHL hockey clubs in the 1996/97 season using data envelopment analysis. The DEA model includes data on player salaries (inputs), league points in the regular season and playoff results (outputs). The result of his research is a calculated efficiency score for each team, which indicates the team's efficiency compared to other teams in the NHL. The resulting efficiency score shows how many league points in the regular season and playoff wins each team could have achieved with the right team composition and correct tactical decisions. The results of the research showed that player salaries do not fully explain the success of teams in the regular season and playoffs. Inefficiency in achieving playoff wins and league points in the regular season with a given roster can be caused by mistakes in player selection or coaching.

Jablonský (2021) examines the relationship between individual player efficiency and team efficiency and models team efficiency as a function of player efficiency. Individual player efficiency is measured using traditional radial and additive Slacks-Based Measure DEA models. The efficiency of teams is then determined by traditional DEA models with variables describing the actual achievements of teams and parallel DEA models that take into account all player positions and the actual performance of teams in the league. The study is based on NHL statistics for the 2019/20 season. The results of the analysis show that actual team performance is not always directly dependent on the individual performance of team members. The biggest deviations occurred with the Florida Panthers and Montreal Canadiens. The best team in the regular season, the Boston Bruins, was ranked 10th by the DEA model. The ranking of the teams derived from the overall efficiency is further away from the actual ranking. The average deviation of the derived ranking from the actual ranking is 4.00. The conclusion is consistent with the fact that a team is always more than the sum of individuals.

In his efficiency analysis, Kahane (2005) attempts to identify sources of inefficiency in NHL clubs using a stochastic frontier production function. Inefficiency in the NHL can be traced in part to differences in coaching ability, team ownership, local sports competition, and management experience. His research concluded that teams with unusually high or very low numbers of French-Canadian players are less efficient. Another source of inefficiency is poor coaching instructions and other factors such as the age of the club. Another possible source of inefficiency in hockey clubs is the type of ownership structure. For example, clubs owned by corporations tend to be more efficient than clubs owned by individuals. Bedford and Baglin (2009) measure team performance during hockey games using regression analysis. Their model is based on direct interaction between two competing teams. They include a performance measure in their model to assess the performance of a team over the course of a match. The research was applied to NHL teams playing in the 2005/06 and 2006/07 seasons. The results of the analysis provided an objective, simple and versatile measure of team performance that would be a valuable evaluative tool for coaches, media and spectators. Weissbock et al. (2013) propose a machine learning approach to predict success in the NHL. The approach combines traditional statistics, such as the number of goals scored and conceded, and performance metrics, such as the number of goals scored in games, to create a classification model. The best results were obtained using neural networks with an accuracy of 59.38%. This model can be used to predict the winner of the playoffs and the winner of the Stanley Cup.

In the Czech environment, hockey efficiency was only examined by the consultancy PwC (2015), which analysed each country's conditions for success at the 2015 World Ice Hockey Championships held in Prague and Ostrava. The research used regression analysis to estimate the so-called PwC point index. This index assesses the historical performance of national teams over the past 20 years at World Cups. Z It takes into account each place in the final ranking. The analysis also takes into account the number of stadiums, the number of registered hockey players, demographic and economic indicators and the average annual temperature in the country. The research confirmed the hypothesis that well-functioning national teams are mostly in countries with a long hockey tradition and popularity. The research also concluded that a country's economic prosperity does not affect the efficiency of national teams. A regression analysis was also used to determine a model-based estimate of team performance based on the included input variables. These values represent the performance that teams should achieve.

It is clear from the above that the literature focuses mainly on the most famous hockey competition - the NHL. No attention is paid to other hockey competitions. Efficiency measurement focuses predominantly on the club level. Researchers pay minimal attention to efficiency at the level of national hockey teams or at the level of individual players. Czech hockey in general and the Czech top hockey league have not received any attention in the literature. The aim of this research is to use data envelopment analysis to help Czech hockey clubs, managers and coaches to evaluate the efficiency of their players. The presented research determines the level of technical efficiency of Czech hockey players using basic methods of data envelopment analysis, determines the ranking of the best players based on their super-efficiency scores and then tries to reveal the main sources of player inefficiency. The models are empirically applied to players of the Czech top hockey competition (i.e. Tipsport extraliga) in the 2021/22 season. The results of the research may be interesting not only for sports managers, coaches and hockey scouts, but also for fans.

METHODS AND DATA

Data

As mentioned in the introduction, the aim of the research is to use data envelopment analysis to determine the level of technical efficiency and then to compare the efficiency of hockey players playing in the Czech top hockey competition, i.e. Tipsport extraliga. Player efficiency was analyzed in the 2021/22 season and was evaluated based on player statistics from the regular season and then from the playoffs. Furthermore, the research seeks to uncover the main sources of player inefficiency from both the regular season and the playoffs.

The data used for research purposes come from the official statistical database of Czech hockey, which is operated by BPA sport marketing a.s. (2022) and eSports.cz, s.r.o. (2022). The database includes a wide range of data related to different levels of hockey competitions. The period under consideration covers the entire 2021/22 season. A total of 15 hockey clubs participated in the Tipsport extraliga in the analyzed 2021/22 season. Namely: Sparta Praha (SPA), Kometa Brno (KOM), Motor České Budějovice (CEB), Bílí Tygři Liberec (LIB), Mladá Boleslav (MLB), Dynamo Pardubice (PCE), Mountfield Hradec Králové (MHK), Verva Litvínov (LIT), Vítkovice Ridera (VIT), Oceláři Třinec (TRI), Rytíři Kladno (KLA), Škoda Plzeň (PLZ), Olomouc (OLO), Berani Zlín (ZLN) and Energie Karlovy Vary (KVA). A total of 437 players who played at least 60 minutes in the 2021/22 regular season were analyzed. In addition, 272 players who played at least 15 minutes in the 2021/22 playoffs were analyzed. Players were divided into three groups according to their game position (goalkeepers, defenders and forwards). Data on individual players was obtained from the website of BPA sport marketing a.s. (2022). These data include mainly ice time, shots on goal, number of goals, number of assists, hits, number of shots blocked by a player, number of games won, save percentage and number of shootouts.

Data Envelopment Analysis

As mentioned in the introduction, the research is aimed at evaluating the efficiency of hockey players in Tipsport extraliga using the data envelopment analysis (further DEA) method. The DEA method generalizes Farrell's (1957) measure of technical efficiency to the case of multiple inputs and multiple outputs. The DEA was first defined by Charnes, Cooper and Rhodes in 1978. It is a method that measures the relative efficiency of so-called decision-making units (further DMUs). DEA is based on linear programming and compares the levels of inputs and outputs of the decision-making unit with those of other DMUs in its peer group (Cooper, 2011). In sports, a DMU can be an athlete, team manager, coach, sports club, etc. In this article, the decision-making units are players of individual hockey clubs playing in the Tipsport extraliga. The DEA method was developed to analyze the relative efficiency of DMUs with heterogeneous inputs and outputs. Therefore, the use of this method seems to be appropriate in the sports industry mainly because a set of different variables can be evaluated (Dlouhý et al., 2018). In this case, the DEA method evaluates the efficiency with which the player can transform his inputs into outputs, i.e., how much outputs the player can achieve given the amount of available inputs (Jablonský and Dlouhý, 2015).

DEA models can be classified according to their orientation into input-oriented, output-oriented and non-oriented models. Input-oriented models help to determine how much inputs need to be reduced to make the evaluated unit efficient. Output-oriented models, on the other hand, help determine by how much outputs need to be increased to make the evaluated unit efficient. In relation to the choice of DEA model orientation, a literature search revealed that the most common orientation used in studies using the DEA method is input orientation. An input-oriented model was also used in this research, with the choice guided primarily by previous literature and the fact that sports clubs generally have more control over their inputs than their outputs. Further subdivision of DEA models is possible based on the nature of the production process. In this case, a distinction can be made between models based on the assumption of constant returns to scale (further CRS), e.g. the CCR model (named after its creators Charnes, Cooper and Rhodes) and models based on the assumption of variable returns to scale (further VRS), e.g. the BCC model (named after its creators Banker, Charnes and Cooper). In the present research, both models are used simultaneously. By using both models, it is possible to identify the main sources of inefficiency of the evaluated DMUs.

From a mathematical point of view, DEA is a technique to obtain information about a given sample of observations in a situation where the production function is not known in advance. An input-oriented CCR model was applied to the obtained data on individual players. The aim of this model is to utilize the minimum level of inputs at the same level of output. The dual form of its mathematical model is formulated using relations (1) and (2). Where λ_j, $j = 1, 2, ..., n$ are weights of all DMUs, $i = 1, 2, ..., m$ and , $k = 1, 2, ..., r$ are slack/surplus variables, is the efficiency score of the DMU_q (Charnes et al., 1978).

$$\text{Minimize} \quad \theta_q \tag{1}$$

$$\begin{aligned} \text{S.t.} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta_q x_{iq}, \quad i = 1,2,\dots,m, \\ & \sum_{j=1}^{n} y_{kj}\lambda_j - s_k^+ = y_{kq}, \quad k = 1,2,\dots,r, \\ & \lambda_j \geq 0, \ \ j = 1,2,\dots,n, \end{aligned} \tag{2}$$

The model described by relations (1) and (2) works under conditions of constant returns to scale. In the literature, the input-oriented CCR model is referred to as CCR-I (Charnes et al., 1978). The CCR-I model makes it possible to calculate the so-called overall technical efficiency (OTE) score. OTE measures efficiencies due to the input/output configuration and as well as the size of operations (Avkiran, 2006). The OTE score is therefore influenced by the so-called scale efficiency. This OTE score is within a range from zero to one, $0 < \text{OTE} \leq 1$. Using the CCR-I model, OTE score were determined for each hockey player. An efficient hockey player gains an OTE score = 1. An OTE score < 1 indicates an inefficient hockey player. The lower the score, the worse the player. It should be noted that the OTE score also helps to identify the source of the technical inefficiency, which can be caused by pure technical inefficiency (PTIE), scale inefficiency (SIE), or both inefficiencies at the same time i.e. overall technical inefficiency (OTIE). The source of such a player's inefficiency may be a lack of game performance, an incorrectly chosen game strategy, or a combination of both.

The input-oriented BCC model was also applied to the obtained data on individual players. The BCC model is based on the assumption of variable returns to scale. The BCC model, originally introduced in Banker et al. (1984), extends the model described by relations (1) and (2) by the convexity condition $\Sigma_{\lambda j} = 1$. The mathematical form of BCC model is as follows:

$$\text{Minimize} \quad \theta_q \tag{3}$$

$$\begin{aligned} \text{S.t.} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta_q x_{iq}, \quad i = 1,2,\dots,m, \\ & \sum_{j=1}^{n} y_{kj}\lambda_j - s_k^+ = y_{kq}, \quad k = 1,2,\dots,r, \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0, \ \ j = 1,2,\dots,n, \end{aligned} \tag{4}$$

In the literature, the input-oriented BCC model described by relations (3) and (4) is referred to as BCC-I (Banker et al., 1984). The BCC-I model allows the calculation of a so-called pure technical efficiency (PTE) score. The PTE score is without the influence of scale efficiency. This PTE score is within a range from zero to one, $0 < \text{PTE} \leq 1$. Using the BCC-I model, PTE score were determined for each hockey player. An efficient hockey player gains a PTE score = 1. PTE score < 1 indicates an inefficient hockey player. The lower the score, the worse the player. The only source of this player's inefficiency is his lack of game performance. Scale efficiency (SE) for each DMU can be obtained by a ratio of OTE score to PTE score (see formula 5). This SE score is within a range from zero to one, 0 < SE ≤ 1. The SE score for each player was determined using relationship (5). An efficient hockey player gained SE score = 1. SE score < 1 indicates an inefficient hockey player. The source of this player's inefficiency may be an incorrectly chosen game strategy. If a DMU is characterized as efficient in the CCR model, it will also be characterized as efficient in the BCC model. However, the reverse is not necessarily true.

$$SE = \frac{OTE}{PTE} \tag{5}$$

It is important to note that all efficient DMUs have an OTE score equal to 1 in the CCR model. Therefore, the efficient DMU cannot be ranked or distinguished using the CCR model. The ability to rank or discriminate efficient DMUs is of theoretical and practical importance. So-called super-efficiency models were formulated to rank DMUs with an efficiency score equal to one. The best known model is that of Andersen and Petersen (1993). In principle, the super-efficiency score for an efficient DMU can take any value greater than or equal to 1. The whole concept of super-efficiency is based on the exclusion of efficient DMUs from the considered set, thus shifting the original efficient frontier. This procedure allows the ranking of efficient DMUs (i.e., the higher the super-efficiency score, the higher the ranking). Inefficient DMUs that are not at the efficiency frontier and whose initial DEA score is less than 1 are not affected by the exclusion of efficient DMUs from the reference set.

Jablonský and Dlouhý (2015) formulate the Andersen and Petersen model (further AP model) with an input-oriented CRS assumption using relations (6) and (7). Its input oriented formulation is very close to the traditional input oriented formulation of CCR-I model, see relations (1) and (2). If the unit under consideration is marked as efficient, then $\theta q^{AP}>1$ applies (Jablonský, 2016). All calculations were performed using MaxDEA software.

$$\text{Minimize} \quad \theta_q^{AP} \tag{6}$$

$$\begin{aligned} \text{S.t.} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta_q^{AP} x_{iq}, \quad i = 1, \dots, m, \\ & \sum_{j=1}^{n} y_{kj}\lambda_j - s_k^+ = y_{kq}, \quad k = 1, \dots, r, \\ & \lambda_j \geq 0, \quad j = 1, \dots, n, j \neq q, \\ & \lambda_q = 0. \end{aligned} \tag{7}$$

In the next step, a suitable number of input/output variables should be selected. According to Cooper (2011), the number of DMUs should be at least two to three times higher than the sum of inputs and outputs. Since hockey players are evaluated, the DEA model will have a sufficiently large number of DMUs with respect to the sum of inputs and outputs. Due to the large number of DMUs evaluated, 9 variables could be selected for the DEA model based on correlation analysis. The CCR-I, BCC-I and AP models are based on a single input variable common to all game positions. The input variable is time on ice (TOI). A player's importance in the team increases every time he is selected for a game and also if he plays a significant amount of game time. The output variables included game statistics relevant to each game position. For defenders, the following output variables were included in the research: number of shots (S), number of goals (G), number of assists (A), hits (H) and the number of blocked shots by the player (BkS). For forwards, the following output variables were included in the research: number of shots (S), number of goals (G) and number of assists (A). For goalkeepers, the following output variables were included in the research: number of matches won (W), save percentage (Sv%) and number of shootouts (SO).

RESEARCH RESULTS

This part of the paper is devoted to the results of an empirical research in which a non-parametric DEA methodology was used. The DMUs in this research are the players of the top Czech hockey competition Tipsport extraliga in the 2021/22 season. Hockey players were divided into three groups according to their game positions. In the first part of the research, players were evaluated on statistical data from the regular season.

DEA – regular season 2021/22

The CCR-I model and the AP super-efficiency model were applied to the statistical data on goalkeepers, defenders and forwards who played at least 60 minutes in the 2021/22 regular season. Subsequently, the BCC-I model was also applied to help identify sources of inefficiency.

Using the CCR-I model, OTE score was determined for all goalkeepers who played at least 60 minutes in the regular season. The average OTE score for all goalkeepers in Tipsport extraliga is around 0.737. The median OTE score is 0.726, based on this value it can be concluded that half of the goalkeepers had worse OTE score and half had better ones. Furthermore, based on the calculated OTE score, it can be stated that out of the 37 goalkeepers evaluated, 5 goalkeepers (13.5%) were efficient. Two efficient goalkeepers were identified in the hockey club Sparta Praha (Július Hudáček and Matěj Machovský). One efficient goalkeeper was identified in Olomouc (Branislav Konrád), Oceláři Třinec (Ondřej Kacetl) and Motor České Budějovice (Jan Strmeň). In order to further classify efficient goalkeepers, the AP super-efficiency model was applied to the data. Table 1 shows the efficient goalkeepers ranked by AP score. The best goalkeeper of the regular season was Július Hudáček (1.1370). Table 1 also shows the mean and median values of the variables included in the DEA.

Table 1. Efficient goalkeepers in the 2021/22 regular season and descriptive statistics of variables

Ranking	Goalkeeper	Club	AP score	TOI	W	Sv%	SO
1.	Július Hudáček	SPA	**1.1370**	425	5	91.67	1
2.	Branislav Konrád	OLO	**1.1196**	1031	9	92.37	3

3.	Ondřej Kacetl	TRI	**1.0121**	1394	17	93.00	2
4.	Jan Strmeň	CEB	**1.0029**	746	9	92.69	1
5.	Matěj Machovský	SPA	**1.0004**	1005	12	92.00	2
Average				920.20	10.40	92.35	1.80
Median				1005.00	9.00	92.37	2.00

For the other clubs, none of the goalkeepers was named as an efficient unit. Thus, a total of 32 goalkeepers (86.5%) can be classified as inefficient units. Relatively good OTE scores (> 0.9) were achieved by goalkeepers Marek Mazanec (0.9750), Jakub Sedláček (0.9639), Marek Schwarz (0.9176) and Filip Novotný (0.9148). The goalkeepers with the lowest OTE scores were Libor Kašík (0.4346), Daniel Huf (0.2639) and Šimon Zajíček (0.1882). These goalkeepers played for Berani Zlín and Verva Litvínov in the 2021/22 season. Berani Zlín ranked 15th, while Verva Litvínov finished in 13th position.

The BCC-I model was also applied to the data to reveal the main sources of goalkeeper inefficiency. The result of this phase of the analysis was the calculation of pure technical efficiency and then scale efficiency. These two values helped to decompose the overall technical efficiency and to identify the main sources of inefficiency. The calculation showed that out of 32 inefficient goalkeepers only five goalkeepers had scale inefficiency (Aleš Stezka, Petr Kváča, Filip Novotný, Jaroslav Janus and Marek Mazanec). The average SE score for these five goalkeepers was 0.8340. Thus, the main source of inefficiency was an incorrectly chosen game strategy. For three goalkeepers, the source of inefficiency is pure technical inefficiency (Gašper Krošelj, Henri Kiviaho and Jan Lukáš). The average PTE score was 0.7063. The main source of inefficiency of these goalkeepers is only their lack of game performance. For the other 24 goalkeepers, the main source of inefficiency is overall technical inefficiency. The result of the research shows that in the regular season of 2021/22, in most cases the reason for the inefficiency of the goalkeepers was a combination of their insufficient game performance and incorrectly chosen game strategy. The average PTE score is 0.6861 and the average SE score is 0.9602. The main problem with these 24 goalkeepers is largely their poor game performance as captured by the PTE score. Table 2 shows the overall summary.

Table 2. Summary of inefficiency results - goalkeepers

		Number	%	Avg. OTE score	Avg. PTE score	Avg. SE score
	Total inefficient	32	86.5	0.6882	0.7370	0.9442
Source	scale inefficiency	5	15.6	0.8340	1.0000	0.8340
	pure technical inefficiency	3	9.4	0.7063	0.7063	1.0000
	overall technical inefficiency	24	75.0	0.6556	0.6861	0.9602

Using the CCR-I model, OTE score was determined for all defenders who played at least 60 minutes in the regular season. The average OTE score of all defenders in Tipsport extraliga is around 0.692. The median OTE score is 0.684, based on this value it can be concluded that half of the defenders achieved worse OTE score and half achieved better. Furthermore, based on the calculated OTE score, it can be stated that of the 146 defenders evaluated, 11 defenders (7.5 %) were considered efficient. Two efficient defenders were identified in Vítkovice Ridera (Alexey Solovyev and Patrik Koch), Sparta Praha (Oleg Pogorishnyi and Maksim Matushkin). One efficient defender was identified in Motor České Budějovice (Ondřej Slováček), Oceláři Třinec (Mikuláš Zbořil), Dynamo Pardubice (Jan Košťálek), Kladno (Jakub Suchánek), Mountfield HK (Bohumil Jank), Kometa Brno (Michal Gulaši) and Verva Litvínov (Jan Strejček). To further classify efficient defenders, the AP super-efficiency model was applied to the data. Table 3 shows the efficient defenders ranked by AP score. The best defender of the regular season was Ondřej Slováček (1.8079). Table 3 shows the mean and median values of the variables included in the DEA.

Table 3. Efficient defenders in the 2021/22 regular season and descriptive statistics of variables

Ranking	Defender	Club	AP score	TOI	H	BkS	S	G	A
1.	Ondřej Slováček	CEB	**1.8079**	106	2	8	9	1	6
2.	Mikuláš Zbořil	TRI	**1.3508**	62	0	6	10	1	0
3.	Jan Košťálek	PCE	**1.2297**	1054	19	49	177	8	33
4.	Alexey Solovyev	VIT	**1.2269**	766	24	33	74	11	17
5.	Patrik Koch	VIT	**1.2158**	1000	94	89	76	2	3
6.	Jakub Suchánek	KLA	**1.1401**	813	59	116	69	2	5
7.	Oleg Pogorishnyi	SPA	**1.0856**	100	4	13	3	1	0
8.	Bohumil Jank	MHK	**1.0590**	665	39	48	72	6	2
9.	Michal Gulaši	KOM	**1.0478**	974	76	136	40	1	7
10.	Maksim Matushkin	SPA	**1.0115**	516	5	14	73	6	14
11.	Jan Strejček	LIT	**1.0044**	504	31	42	36	4	4
Average				596.36	32.09	50.36	58.09	3.91	8.27
Median				665.00	24.00	42.00	69.00	2.00	5.00

For the other clubs, none of the defenders was named as an efficient unit. Thus, a total of 135 defenders (92.5%) can be classified as inefficient units. Relatively good OTE scores (> 0.9) were achieved by defenders Richard Nedomlel (0.9942), Rhett Holland (0.9747), Kevin Tansey (0.9710), David Štich (0.9667), Marian Adámek (0.9235), Daniel Gazda (0.9202), Jiří Ondrušek (0.9189), Tomáš Kundrátek (0.9184) and Jakub Teper (0.9064). The defenders with the lowest OTE score were Patrik Fajmon (0.3174), Šimon Groch (0.3389) and Hakon Nilsen (0.3570).

The BCC-I model was also applied to the data to reveal the main sources of defender inefficiency. The calculation showed that 11 of 135 inefficient defenders had scale inefficiency (Aaron Irving, Daniel Gazda, Jakub Michálek, Jan Štencel, Jiří Ondrušek, Kevin Tansey, Peter Čerešňák, Richard Nedomlel, Tadeáš Talafa, Tomáš Kundrátek and Vojtěch Riedl). The average SE score for these defenders was 0.8192. The main source of inefficiency was an inappropriately chosen game strategy. Pure technical inefficiency was not the source of inefficiency for any of the defenders. For the remaining 124 defenders, the main source of inefficiency is overall technical inefficiency. The result of the research thus shows that in the regular season, in most cases, the cause of the defenders' inefficiency was a combination of their poor game performance and incorrectly chosen game strategy. The average PTE score is 0.6922 and the average SE score is 0.9462. The main problem with these defenders is largely their poor game performance captured by the PTE score. The overall summary is shown in Table 4.

Table 4. Summary of inefficiency results - defenders

		Number	%	Avg. OTE score	Avg. PTE score	Avg. SE score
	Total inefficient	135	92.5	0.6667	0.7173	0.9358
Source	scale inefficiency	11	8.1	0.8192	1.0000	0.8192
	overall technical inefficiency	124	91.9	0.6531	0.6922	0.9462

Using the CCR-I model, OTE score was determined for all forwards who played at least 60 minutes in the regular season. The average OTE score of all forwards in Tipsport extraliga is around 0.5508. The median OTE score is 0.5292, based on this value it can be concluded that half of the forwards achieved worse OTE scores and half achieved better ones. Furthermore, based on the calculated OTE score, it can be stated that out of the 254 forwards evaluated, 4 forwards (1.6%) were considered efficient. Efficient forwards were marked in clubs: Sparta Praha (David Tomášek), Škoda Plzeň (Michal Bulíř), Dynamo Pardubice (Robert Říčka) and Oceláři Třinec (Erik Hrňa). In order to further classify the efficient forwards, the AP super-efficiency model was applied to the data. Table 5 shows the efficient forwards ranked by AP score. The best forward in the regular season was David Tomášek (1.4013). Table 5 also shows the mean and median values of the variables included in the DEA.

Table 5. Efficient forwards in the 2021/22 regular season and descriptive statistics of variables

Ranking	Forward	Club	AP score	TOI	S	G	A
1.	David Tomášek	SPA	**1.4013**	125	20	4	7
2.	Michal Bulíř	PLZ	**1.0942**	977	204	27	29
3.	Robert Říčka	PCE	**1.0549**	807	149	27	20
4.	Erik Hrňa	TRI	**1.0497**	402	60	14	13
Average				577.75	108.25	18.00	17.25
Median				604.50	104.50	20.50	16.50

For the other clubs, none of the forwards was named as an efficient unit. Thus, a total of 250 forwards (98.4%) can be classified as inefficient units. Relatively good OTE scores (> 0.9) were achieved by forwards Filip Chlapík (0.9774), Matěj Blümel (0.9483), Tomáš Záborský (0.9464), Michal Řepík (0.9456), Ahti Oksanen (0.9353), Dominik Lakatoš (0.9267) and Peter Mueller (0.9183). The forwards with the lowest OTE value were Šimon Frömel (0.1430), Jiří Novotný (0.2298) and František Gerhát (0.2362).

The BCC-I model was also applied to the data to reveal the main sources of offensive inefficiency. The calculation showed that out of 250 inefficient forwards only 8 forwards had scale inefficiency (Daniel Kurovský, Filip Chlapík, Martin Štohanzl, Peter Mueller, Samuel Bitten, Tomáš Záborský, Tor Erik Eriksson Immo and Vlastimil Dostálek). The average SE score for these forwards was 0.7761. Thus, the main source of inefficiency was an incorrectly chosen game strategy. Pure technical inefficiency was not the source of inefficiency for any of the forwards. For the other 242 forwards, the main source of inefficiency is overall technical inefficiency. The result of the research thus shows that in the regular season 2021/22, in most cases the reason for the inefficiency of the forwards was a combination of their underperformance and incorrectly chosen game strategy. The average PTE score is 0.5953 and the average SE score is 0.9082. The main problem with these forwards is their poor game performance captured by the PTE score. The overall summary is shown in Table 6.

Table 6. Summary of inefficiency results - forwards

		Number	%	Avg. OTE score	Avg. PTE score	Avg. SE score
	Total inefficient	250	98.4	0.5436	0.6082	0.9040
Source	scale inefficiency	8	3.2	0.7761	1.0000	0.7761
	overall technical inefficiency	242	96.8	0.5359	0.5953	0.9082

Figure 1 compares the sources of inefficiency for each type of game position. It is clear from Figure 1 that scale inefficiency is not as common a problem for players. These players could be helped by a change of club (trade) as they most likely don't suit the set style of play of the team or the strategy of the coaches. Figure 1 also shows that overall technical inefficiency prevails for all game positions. For these players it is necessary to work mainly on performance development in combination with a possible change of game strategy. Only for some goalkeepers pure technical inefficiency was detected by the research. This can be caused by an avalanche effect, where poor play by the forwards affects the defenders, who in turn negatively affect the goalkeepers.

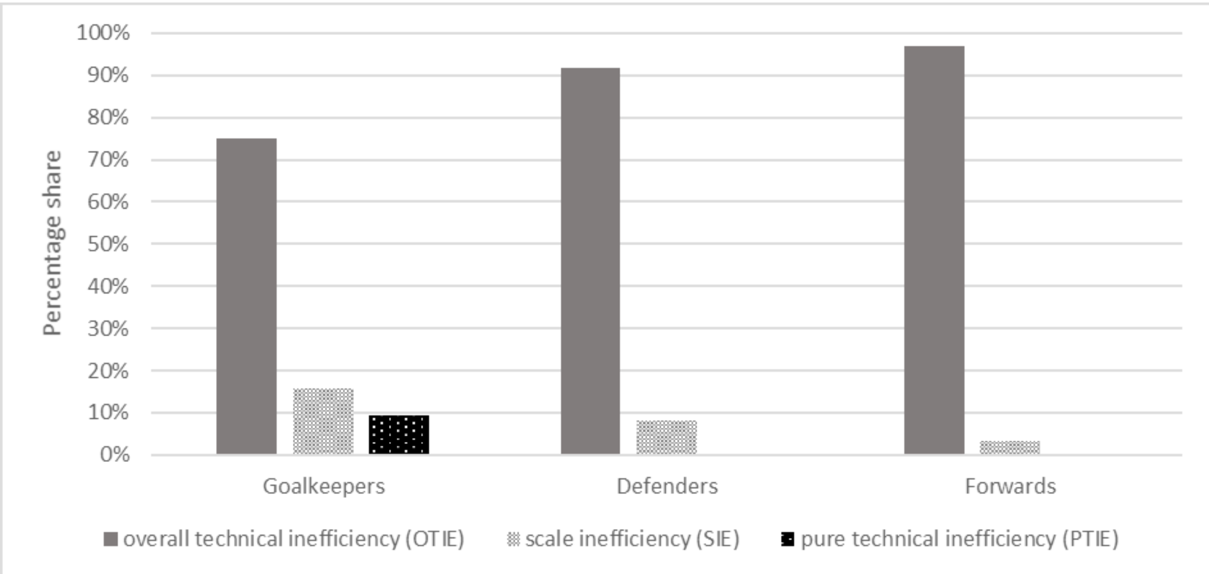

Figure 1. Comparison of sources of inefficiency within individual game positions (regular season)

DEA – 2021/22 playoffs

The CCR-I model, an AP super-efficiency model, was applied to the statistical data on goalkeepers, defenders and forwards who played at least 15 minutes in the 2021/22 playoffs. Subsequently, the BCC-I model was also used to help identify sources of inefficiency.

Using the CCR-I model, OTE score was determined for all goalkeepers who played at least 15 minutes in the playoffs. The average OTE score for all goalkeepers in the playoffs is around 0.569. The median OTE score is 0.532, based on this value it can be concluded that half of the goalkeepers achieved worse OTE scores and half achieved better ones. Furthermore, based on the calculated OTE score, it can be stated that out of the 21 goalkeepers evaluated, 4 goalkeepers (19.05%) were efficient. Two efficient goalkeepers were identified in Oceláři Třinec (Marek Mazanec and Ondřej Kacetl). One efficient goalkeeper was identified in Motor České Budějovice (Jan Strmeň) and Škoda

Plzeň (Miroslav Svoboda). To further classify the efficient goalkeepers, the AP super-efficiency model was applied to the data. Table 7 shows the efficient goalkeepers ranked by AP score. The best goalkeeper in the playoffs was Miroslav Svoboda (2.1733). Table 7 also shows the mean and median values of the variables included in the DEA.

Table 7. Efficient goalkeepers in the 2021/22 play-off and descriptive statistics of variables

Ranking	Goalkeeper	Club	AP score	TOI	W	Sv%	SO
1.	Miroslav Svoboda	PLZ	**2.1733**	119	1	94,74	1
2.	Marek Mazanec	TRI	**1.2798**	128	2	95,24	0
3.	Jan Strmeň	CEB	**1.1429**	15	0	66,67	0
4.	Ondřej Kacetl	TRI	**1.0982**	733	10	93,75	3
Average				248.75	3.25	87.60	1.00
Median				123.50	1.50	94.25	0.50

For the other clubs, none of the goalkeepers was identified as an efficient unit. Thus, a total of 17 goalkeepers (80.95 %) can be classified as inefficient units. Other goalkeepers had OTE values lower than 0.88. The goalkeepers with the lowest OTE were Štěpán Lukeš (0.1102) and Henri Kiviaho (0.1338).

The BCC-I model was also applied to the data to reveal the main sources of goalkeeper inefficiency in the playoffs. The result of this phase of the analysis was the calculation of pure technical efficiency and then scale efficiency. These two values helped to decompose the overall technical efficiency and to identify the main sources of inefficiency. The calculation showed that out of 17 inefficient goalkeepers only two goalkeepers had scale inefficiency (Jaroslav Pavelka and Pavel Jekel). The average SE score for these two goalkeepers was 0.7260. Thus, the main source of inefficiency was an incorrectly chosen game strategy. For two goalkeepers, the source of inefficiency is pure technical inefficiency (Branislav Konrád and Marek Čiliak). The average PTE score was 0.5633. The main source of inefficiency of these goalkeepers is only their lack of game performance. For the remaining 13 goalkeepers, the main source of inefficiency is overall technical inefficiency. The result of the research thus shows that in the playoffs, in most cases the reason for the inefficiency of goalkeepers was a combination of their poor game performance and incorrectly chosen game strategy. The average PTE score is 0.5424 and the average SE score is 0.7508. So the main problem with these goalkeepers is rather their poor game performance captured by the PTE score. The overall summary is presented in Table 8.

Table 8. Summary of inefficiency results – goalkeepers

		Number	%	Avg. OTE score	Avg. PTE score	Avg. SE score
	Total inefficient	17	80.95	0.4672	0.5987	0.7772
Source	scale inefficiency	2	11.8	0.7260	1.0000	0.7260
	pure technical inefficiency	2	11.8	0.5633	0.5633	1.0000
	overall technical inefficiency	13	76.5	0.4126	0.5424	0.7508

Using the CCR-I model, OTE score was determined for all defenders who played at least 15 minutes in the playoffs. The average OTE score for defenders in the playoffs is around 0.698. The median OTE score is 0.659, based on this value it can be concluded that half of the defenders achieved worse OTE scores and half achieved better ones. Furthermore, based on the calculated OTE score, it can be stated that out of the 86 defenders evaluated, 10 defenders (11.6%) were considered efficient. Two efficient defenders were identified in Vítkovice Ridera (Petr Gewiese and Patrik Koch), Mountfield HK (Filip Pavlík and Petr Kalina), Škoda Plzeň (Vladimír Kremláček and Peter Čerešňák) and Kometa Brno (Marek Ďaloga and Radek Kučeřík). One efficient defender was identified in Dynamo Pardubice (Jan Košťálek) a Sparta Praha (Tomáš Pavelka). To further classify the efficient defenders, the AP super-efficiency model was applied to the data. Table 9 shows the efficient defenders ranked by AP score. The best defender of the playoffs was Filip Pavlík (1.4039). Table 9 also shows the mean and median values of the variables included in the DEA.

Table 9. Efficient defenders in the 2021/22 play-off and descriptive statistics of variables

Ranking	Defender	Club	AP score	TOI	H	AB	S	G	A
1.	Filip Pavlík	MHK	**1.4039**	114	3	4	21	0	1
2.	Petr Gewiese	VIT	**1.3768**	23	2	5	1	0	0
3.	Vladimír Kremláček	PLZ	**1.3389**	30	1	5	3	0	1
4.	Patrik Koch	VIT	**1.3380**	125	13	8	14	0	2
5.	Petr Kalina	MHK	**1.2992**	70	3	5	7	1	1
6.	Peter Čerešňák	PLZ	**1.1741**	130	5	9	12	1	4
7.	Marek Ďaloga	KOM	**1.1463**	99	0	8	11	1	3
8.	Jan Košťálek	PCE	**1.1169**	128	1	4	17	0	4
9.	Radek Kučeřík	KOM	**1.1122**	95	6	15	11	0	0
10.	Tomáš Pavelka	SPA	**1.0448**	268	2	9	21	4	2
Average				108.20	3.60	7.20	11.80	0.70	1.80
Median				106.50	2.50	6.50	11.50	0.00	1.50

For the other clubs, none of the defenders was identified as an efficient unit. Therefore, 76 defenders (88.4%) can be classified as inefficient units. Defenders Dominik Graňák (0.9983), Karel Nedbal (0.9862), Jeremie Blain (0.9576), David Škůrek (0.9432), David Němeček (0.9226), Jan Zahradníček (0.9185) and Tomáš Černý (0.9125) had relatively good OTE scores (> 0.9). The defenders with the lowest OTE scores were David Moravec (0.0029), David Kvasnička (0.2421) and Mitchell Fillman (0.3272).

The BCC-I model was also applied to the data to reveal the main sources of inefficiency. The calculation showed that out of 76 inefficient defenders, 16 defenders had scale inefficiency. The average SE score for these defenders was 0.8514. The main source of inefficiency may have been an incorrectly chosen game strategy. Pure technical inefficiency was not the source of inefficiency for any of the defenders. For the remaining 60 defenders, the main source of inefficiency is overall technical inefficiency. The result of the research thus shows that in the playoffs, the cause of the inefficiency of the defenders was in most cases a combination of their poor game performance and incorrectly chosen game strategy. The average PTE score is 0.6966 and the average SE score is 0.8577. The main problem with these defenders is largely their poor game performance captured by the PTE score. The overall summary is shown in Table 10.

Table 10. Summary of inefficiency results – defenders

		Number	%	Avg. OTE score	Avg. PTE score	Avg. SE score
	Total inefficient	76	88.4	0.6578	0.7605	0.8564
Source	scale inefficiency	16	21.1	0.8514	1.0000	0.8514
	overall technical inefficiency	60	78.9	0.6062	0.6966	0.8577

Using the CCR-I model, OTE score was determined for all forwards who played at least 15 minutes in the playoffs. The average OTE score for all forwards in the playoffs is around 0.5015. The median OTE score is 0.4844, based on this value it can be concluded that half of the forwards achieved worse OTE scores and half achieved better ones. Furthermore, based on the calculated OTE score, it can be stated that out of the 165 forwards evaluated, 6 forwards (3.6 %) were considered efficient. Efficient forwards were identified in Sparta Praha (David Tomášek), Škoda Plzeň (Michal Bulíř), Energie Karlovy Vary (Tomáš Redlich), Mladá Boleslav (Matyáš Kantner), Kometa Brno (Peter Mueller) and Motor České Budějovice (Daniel Voženílek). In order to further classify the efficient forwards, the AP super-efficiency model was applied to the data. Table 11 shows the efficient forwards ranked by AP score. The best forward in the playoffs was Tomáš Redlich (1.9429). Table 11 also shows the mean and median values of the variables included in the DEA.

Table 11. Efficient forwards in the 2021/22 play-off and descriptive statistics of variables

Ranking	Forward	Club	AP score	TOI	S	G	A
1.	Tomáš Redlich	KVA	**1.9429**	20	3	0	2
2.	Matyáš Kantner	MLB	**1.1744**	43	9	2	0
3.	Michal Bulíř	PLZ	**1.1627**	95	27	2	2
4.	Peter Mueller	KOM	**1.1087**	90	22	3	3
5.	Daniel Voženílek	CEB	**1.1001**	112	13	5	2
6.	David Tomášek	SPA	**1.0088**	184	35	6	7
Average				90.67	18.17	3.00	2.67
Median				92.50	17.50	2.50	2.00

For the other clubs, none of the forwards was identified as an efficient unit. Thus, a total of 159 forwards (96.4 %) can be classified as inefficient units. Forwards Ondřej Beránek (0.9613), Michal Birner (0.9287), Michal Kunc (0.9205) and Tomáš Vondráček (0.9089) had relatively good OTE scores (> 0.9). The forwards with the lowest OTE score were Vojtěch Lednický (0.0009), Tomáš Knotek (0.0690) and Vít Jiskra (0.1466).

The BCC-I model was also applied to the data to reveal the main sources of inefficiency of the forwards. The calculation showed that out of 159 inefficient forwards only five forwards had scale inefficiency (Filip Chlapík, Michal Řepík, Miloš Kelemen, Tomáš Filippi and Tomáš Urban). The average SE score for these forwards was 0.8172. Thus, the main source of inefficiency was an incorrectly chosen game strategy. For 20 forwards, pure technical inefficiency was the source of inefficiency. The average PTE score was 0.4693. The main source of the inefficiency of these forwards is only their lack of game performance. For the other 134 forwards, the main source of inefficiency is overall technical inefficiency. The result of the research shows that in the playoffs of the 2021/22 season, in most cases the reason for the inefficiency of the forwards was a combination of their lack of game performance and incorrectly chosen game strategy. The average PTE score is 0.5741 and the average SE score is 0.8301. The main problem with these forwards is their poor game performance captured by the PTE score. The overall summary is shown in Table 12.

Table 12. Summary of inefficiency results - forwards

		Number	%	Avg. OTE score	Avg. PTE score	Avg. SE score
	Total inefficient	159	96.4	0.4826	0.5743	0.8511
Source	scale inefficiency	5	3.1	0.8172	1.0000	0.8172
	pure technical inefficiency	20	12.6	0.4693	0.4693	1.0000
	overall technical inefficiency	134	84.3	0.4722	0.5741	0.8301

Figure 2 compares the sources of inefficiency for each type of game position in the playoffs. Figure 2 shows that the main source of inefficiency is overall technical inefficiency. For goaltenders, the sources of inefficiency changed only slightly compared to the regular season. There was a change in the defenders, where the proportion of scale inefficiency increased slightly compared to the regular season. There was also a change in the group of forwards. The playoffs saw a very significant pure technical inefficiency compared to the regular season (increase from 0 to 20 forwards). For these players, the main cause of inefficiency was probably performance or physical unpreparedness for the playoffs. A possible reason for this phenomenon is the different length of the two Tipsport extraliga periods. In the playoffs, players' performance or physical deficiencies, which are not so noticeable in the regular season (played for 56 rounds), were more evident.

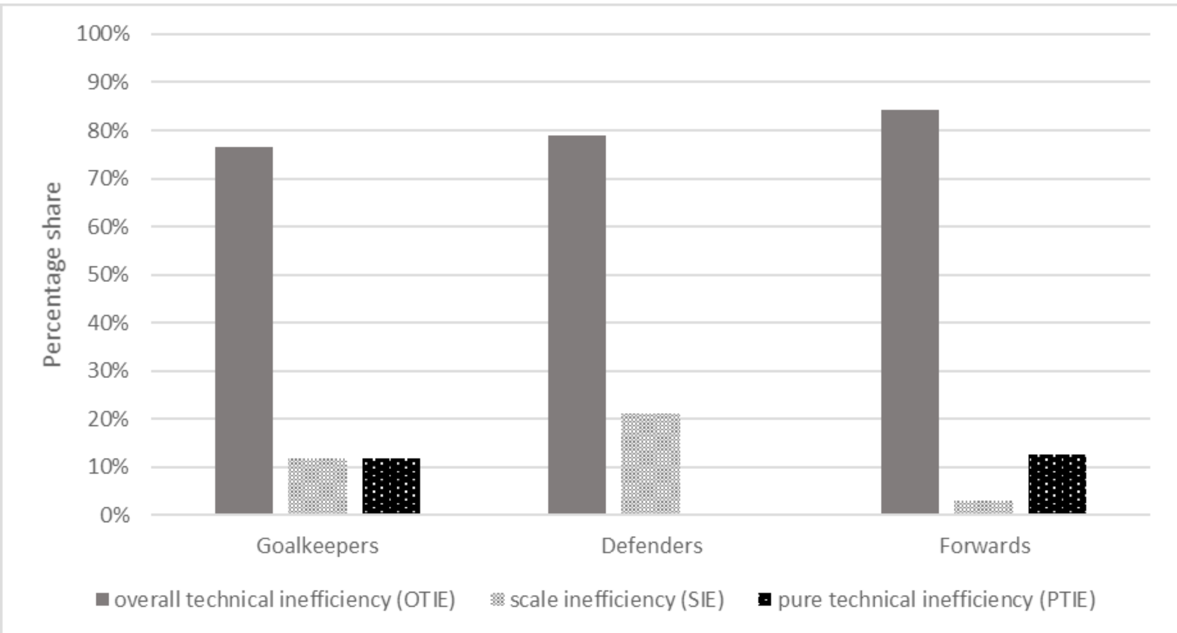

Figure 2. Comparison of sources of inefficiency within individual game positions (play-off)

DISCUSSION

Considering the current economic and financial situation of hockey clubs, it is more than necessary to know how efficiently a club uses its financial and human resources. The analysis of the efficiency of hockey players is an important research topic in the field of player evaluation and selection, as well as the composition of hockey teams for the new season. The analysis enables the identification of sources of inefficiency and helps to find corrective measures to avoid wasting resources. The presented research uses basic methods of data envelopment analysis to determine the level of technical efficiency of Czech hockey players, ranks the best players based on their super-efficiency scores, and then tries to uncover the main sources of player inefficiency.

The results showed that the relative frequency of efficient goalkeepers is higher in the playoffs (19.05%) than in the regular season (13.5 %). On the other hand, both the average and median OTE scores were lower for goalkeepers in the playoffs than in the regular season. For goalkeepers, all sources of inefficiency were identified. For most goalkeepers, overall technical inefficiency prevailed. The differences in the structure of sources of inefficiency in the regular season and playoffs were minimal. Research on defenders has shown that the relative frequency of efficient defenders is higher in the playoffs (11.6 %) than in the regular season (7.5 %). The average and median OTE scores in the playoffs and regular season showed only minimal differences. For defenders, only two sources of inefficiency were identified - overall technical inefficiency and scale inefficiency. Overall technical inefficiency was predominant. It can also be concluded that in the playoffs there was a higher proportion of defenders suffering from scale inefficiency. The results also showed that the relative frequency of efficient forwards is higher in the playoffs (3.6%) than in the regular season (1.6%). Both the average and median OTE scores were slightly lower in the playoffs. The structure of sources of inefficiency differed in the regular season and the playoffs. No forward showed pure technical inefficiency in the regular season. On the other hand, 12.6% of forwards suffered from pure technical inefficiency in the playoffs. In the short-term part of the competition, the players‘ performance or physical deficiencies were more evident.

The research reaches similar conclusions to Kahane (2005), who states that inefficiency in the NHL can be traced in part to differences in coaching ability and management experience. This source of player inefficiency has also been confirmed in the Czech Tipsport extraliga. The inefficiency resulting from poor coaching decisions was confirmed for all groups of players and was captured by scale inefficiency. However, the main source of player inefficiency in the Czech league was overall technical inefficiency. Most players need to work on performance development in combination with a change in game strategy. Performance is not as big a problem in the NHL as it is in the Czech league due to the very wide player base.

Unfortunately, the Czech Tisport extraliga failed to prove the conclusions of Kuosmanen (1998), who claims that player salaries do not fully explain the success of teams in the regular season and playoffs. Player salary could not be included in the models in the Czech hockey environment due to unavailability of data and the research had to be based on playing time.

CONCLUSION

The aim of this research was to determine the level of technical efficiency of hockey players in the Czech Tipsport extraliga. Furthermore, to create a ranking of the best hockey players and to reveal the main sources of player inefficiency.

The presented methodology allows to use a comparative system to evaluate the efficiency of players and teams at both club and national level. It can primarily help sports managers of clubs to gain knowledge about the performance of their players and subsequently contribute to improving their performance or help with the composition of players for the new season. Along with general knowledge and experience, coaches can also take DEA efficiency analysis into account for team development and for tactical preparation of goalkeepers, defenders and forwards for matches. This evaluation model could contribute to a more accurate differentiation of the quality of individual players and to the evaluation of the overall efficiency of the team after the end of the season, if information about players' salaries is included. The quality of individual players is also important for hockey scouts who can also use the DEA method. The DEA is based on quantitative variables and can be used as a complementary method to attributes such as player's personality, behaviour, player's game thinking, skating attitude, stability, skating economy, speed and agility.

For better and more accurate results, it would be desirable to supplement the data envelopment analysis with another input variable, which would be the players' salary. Currently, however, player salaries are not a publicly available variable and it is difficult for an outside analyst to determine this data. Czech hockey differs significantly from other hockey competitions when it comes to player salary disclosure. For example, the NHL or Scandinavian hockey competitions are more transparent in this respect and provide data on players' salaries to the wider public. In the future, this input variable would make it possible to make the efficiency results of individual players and subsequently of entire teams more precise. For example, players who have average or below-average salaries can achieve very interesting game results and can be identified as an efficient unit in terms of the DEA model. Such a player could be an interesting acquisition for the club when building the team for the new season. On the other hand, inefficient players with above-average financial compensation represent a waste of the club's financial resources and contribute to a decrease in the overall efficiency of the team. Other output variables can also be added to the research, such as face-offs won, penalty minutes, plus-minus statistics, etc. Future research in the field of ice hockey can also target individual players' game or technical skills and include variables in the DEA model that are only related to offensive or defensive activities.

The presented methodology can also be applied to Czech hockey players playing at the national team level. Then compare the results with foreign rivals. It is also important to mention that the research was applied only for one season. Future research should focus on determining the technical efficiency of players and clubs with more seasons. Based on a longer time series, it should be clear how the club responded to the calculated efficiency and whether it tried to make changes in subsequent seasons. For this purpose, the data envelopment analysis would be complemented by the Malmquist index, which helps with tracking technical efficiency in a time series.

REFERENCES

Andersen T. R., & Sharp, G. P. (1997). A new measure for baseball hitters using DEA. *Annals of Operations Research*, *73*(0), 141–155. http://dx.doi.org/10.1023/A:1018921026476

Andersen, P., & Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science*, *39*(10), 1261–1264. https://doi.org/10.1287/mnsc.39.10.1261

Avkiran, N. K. (2006). *Productivity analysis in the services sector with data envelopment analysis*. University of Queensland Business School.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science*, *30*(9), 1078–1092. https://doi.org/10.1287/mnsc.30.9.1078

Bedford, A., & Baglin, J. (2009). Evaluating the performance of an ice hockey team using interactive phases of play. *IMA Journal of Management Mathematics*, *20*(2), 159–166. http://dx.doi.org/10.1093/imaman/dpn019

BPA sport marketing a.s. (2022). *Statistiky: Tipsport elh: Hokej.cz - web českého hokeje*. Retrieved 20 November, 2022. Available from https://www.hokej.cz/tipsport-extraliga/stats-center?season=2021&competition=6877&team=&age=&state=&timeFrom=&yearFrom=&yearTo=&position=

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, *2*(6), 429–444. https://doi.org/10.1016/0377-2217(78)90138-8

Collier, T., Johnson, A. L., & Ruggiero, J. (2011). Measuring Technical Efficiency in Sports. *Journal of Sports Economics*, *12*(6), 579–598. https://doi.org/10.1177/1527002510391582

Cooper, W. W., Ruiz, J. L., & Sirvent, I. (2009). Selecting non-zero weights to evaluate effectiveness of basketball players with DEA. *European Journal of Operational Research*, *195*(2), 563–574. https://doi.org/10.1016/j.ejor.2008.02.012

Cooper, W. W., Ramón, N., Ruiz, J. L., & Sirvent, I. (2011). Avoiding large differences in weights in cross-efficiency evaluations: application to the ranking of basketball player. *Journal of centrum cathedra: The Business and Economics Research Journal*, *4*(2), 197–215. http://dx.doi.org/10.7835/jcc-berj-2011-0058

Cooper, W. W. (2011). *Handbook on data envelopment analysis*. Springer.

Dlouhý, M., Jablonský, J., & Zýková, P. (2018). *Analýza obalu dat*. Professional Publishing.

Espitia-Escue, M., & García-Cebrián, L. Í. I. (2004). Masuring the Efficiency of Spanish First-Division Soccer Teams. *Journal of Sports Economics*, *5*(4), 329–346. https://doi.org/10.1177/1527002503258047

eSports.cz, s.r.o. (2022). *Hokej - Onlajny*. Retrieved 1 November, 2022. Available from https://www.onlajny.com/hokej

Farrell, M. J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society*, *120*(3), 253–281. https://doi.org/10.2307/2343100

Fizel, J. L., & D'Itri, M. (1996). Estimating managerial efficiency: The case of college basketball coaches. *Journal of Sport Management*, *10*(4), 435–445. https://doi.org/10.1123/jsm.10.4.435

Fried, H. O., Lambrinos, J., & Tyner, J. (2004). Evaluating the performance of professional golfers on the PGA, LPGA and SPGA tours. *European Journal of Operational Research*, *154*(2), 548–561. https://doi.org/10.1016/S0377-2217(03)00188-7

Guzmán, I., & Morrow, S. (2007). Measuring efficiency and productivity in professional football teams: evidence from the English Premier League. *Central European Journal of Operations Research*, *15*(4), 309–328. https://doi.org/10.1007/s10100-007-0034-y

Guzmán-Raja, I., & Guzmán-Raja, M. (2021). Measuring the efficiency of football clubs using data envelopment analysis: Empirical evidence from Spanish professional football. *SAGE Open*, *11*(1), 1–13. https://doi.org/10.1177/2158244021989257

Haas, D. J. (2003). Productivity efficiency of English football teams: A data envelopment analysis approach. *Managerial and Decision Economics*, *24*(5), 403–410. https://doi.org/10.1002/mde.1105

Haas, D. J., Kocher, M. G., & Sutter, M. (2004). Measuring efficiency of German football teams by data envelopment analysis. *Central European Journal of Operations Research and Economics*, *12*, 251–268.

Hadley, L., Poitras, M., Ruggiero, J., & Knowles, S. (2000). Performance evaluation of National Football League teams. *Managerial and Decision Economics*, *21*(2), 63–70. http://dx.doi.org/10.1002/1099-1468(200003)21:2<63::AID-MDE964>3.0.CO;2-O

Halkos, G., & Tzeremes, N. (2012). *Evaluating professional tennis players' career performance: A Data Envelopment Analysis approach*. MPRA Paper 41516. University Library of Munich.

Jablonský, J. (2016). Ranking models in Data Envelopment Analysis. *Trendy v podnikání*, *6*(4), 36–42.

Jablonský, J. (2021). Individual and team efficiency: a case of the National Hockey League. *Central European Journal of Operations Research*, *30*(2), 479–494. https://doi.org/10.1007/s10100-021-00775-0

Jablonský, J., & Dlouhý, M. (2015). *Modely hodnocení efektivnosti a alokace zdrojů*. Professional Publishing.

Kahane, L. H. (2005). Production Efficiency and Discriminatory Hiring Practices in the National Hockey League: A Stochastic Frontier Approach. *Review of Industrial Organization*, *27*(1), 47–71. http://dx.doi.org/10.1007/s11151-005-4400-4

Kian, I. (2009). Profitability and marketability efficiency of large banks-an application of data envelopment analysis. *Journal of Business*, *96*(9), 101–113.

Kirschstein, T., & Liebscher, S. (2018). Assessing the market values of soccer players - a robust analysis of data from German 1. and 2. Bundesliga. *Journal of Applied Statistics*, *46*(7), 1336–1349. https://doi.org/10.1080/02664763.2018.1540689

Kuosmanen, T. (1998). *Efficiency of Hockey Teams in NHL*. Helsinki School of Economics and Business Administration.

Mazur, M. (1994). Evaluating the relative efficiency of baseball players. In A. Charnes, W. W. Cooper, A. Lewin, & L. Seiford (Eds.), *Data envelopment analysis: Theory, methodology and applications* (pp. 369-391). Kluwer.

Palafox-Alcantar, P. G., & Vargas-Hernández, J. G. (2015). Measuring the efficiency of the 32 franchises in the NFL during the 2014 season. *Journal of Sports Economics & Management*, *5*(1), 37–53.

Porter, P. K., & Scully, G. W. (1982). Measuring managerial efficiency: The case of baseball. *Southern Economic Journal*, *48*(3), 642–650. https://doi.org/10.2307/1058656

PWC. (2015). *Ice Hockey National Team Performance Model*. PricewaterhouseCoopers

Ramón, N., Ruiz, J. L., & Sirvent, I. (2012). Common sets of weights as summaries of DEA profiles of weights: With an application to the ranking of professional tennis players. *Expert Systems with Applica-tions*, *39*(5), 4882–4889. https://doi.org/10.1016/j.eswa.2011.10.004

Ruggiero, J., Hadley, L., & Gustafson, E. (1996). Technical efficiency in major league baseball. In J. Fizel, E. Gustafson & L. Hadley (Eds.), *Baseball economics: Current research* (pp. 191-200). Westport, CT: Praeger.

Suk, Y. K. (2014). Selecting the Batters of National Baseball Squad using Data Envelopment Analysis. *Journal of the Korea Academia-Industrial cooperation Society*, *15*(1), 165–172. https://doi.org/10.5762/kais.2014.15.1.165

Weissbock, J., Viktor, H. L., & Inkpen, D. (2013). *Use of Performance Metrics to Forecast Success in the National Hockey League*. ECML/PKDD

Zak, T. A., Huang, C. J., & Siegfried, J. J. (1979). Production efficiency - Case of professional basketball. *The Journal of Business*, *52*(3), 379–392. https://doi.org/10.1086/296053

Zambom-Ferraresi, F., García-Cebrián, L. I., Lera-López, F., & Iráizoz, B. (2015). Performance evaluation in the UEFA Champions League. *Journal of Sports Economics*, *18*(5), 448–470. https://doi.org/10.1177/1527002515588135

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